PP46902

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$\sum_{cyc} \frac{1}{c(s-a)(s-b)} = \frac{s^2+r^2-8Rr}{4sRr^3}$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$\sum_{cyc} \frac{1}{c(s-a)(s-b)} = \sum_{cyc} \frac{ab}{abc(s-a)(s-b)}$$

$$= \frac{1}{abc} \sum_{cyc} \frac{ab}{(s-a)(s-b)} =$$

$$= \frac{1}{abc(s-a)(s-b)(s-c)} \sum_{cyc} ab(s-c) =$$

$$= \frac{s}{abc \cdot F^2} \left(s \sum_{cyc} ab - 3abc \right) =$$

$$= \frac{s}{4RF^3} \cdot (s \cdot (s^2+r^2+4Rr) - 12Rrs) =$$

$$= \frac{s^2}{4Rr^3s^3} \cdot s(s^2+r^2+4Rr-12Rr) =$$

$$= \frac{s^2+r^2-8Rr}{4Rsr^3}$$

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Mathematics Department, National Economic College "Theodor Costescu", Drobeta Turnu - Severin, Romania

Email address: dansitaru63@yahoo.com