PP47286

MIHÁLY BENCZE - ROMANIA

In any ΔABC holds:

$$\sum_{cyc}\frac{1}{h_a r_a}=\frac{4R+r}{s^2 r}$$

Solution by Daniel Sitaru and Claudia Nănuţi.

$$\sum_{cyc}\frac{1}{h_a r_a}=\sum_{cyc}\frac{1}{\frac{2F}{a}\cdot\frac{F}{s-a}}=\frac{1}{2F^2}\sum_{cyc}a(s-a)=$$

$$=\frac{1}{2F^2}\Big(s\sum_{cyc}a-\sum_{cyc}a^2\Big)=$$

$$=\frac{1}{2F^2}(s\cdot\sum_{cyc}a-\sum_{cyc}a^2\Big)=$$

$$=\frac{1}{2F^2}(s\cdot 2s-2(s^2-r^2-4Rr))=$$

$$=\frac{1}{2F^2}(2s^2-2s^2+2r^2+8Rr)=$$

$$=\frac{1}{2r^2s^2}\cdot(2r^2+8Rr)=\frac{2r(4R+r)}{2r^2s^2}=\frac{4R+r}{s^2r}$$

□

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