PP47382

MIHÁLY BENCZE - ROMANIA

Compute:

$$\lim_{x\to 0}\frac{1}{x^{n+1}}\left(\int_0^x \sin^n t dt - x\right)$$

Solution by Daniel Sitaru and Claudia Nănuţi.
Let be $f:[0,\infty)\to\mathbb{R}; f(t)=\sin^n t; n\in\mathbb{N}$
Let be $F:[0,\infty)\to\mathbb{R}$ such that $F'(t)=f(t)$ for any $t>0$.

$$\lim_{x\to 0}\frac{1}{x^{n+1}}\left(\int_0^x \sin^n t dt - x\right)=\lim_{x\to 0}\frac{1}{x^{n+1}}\left(\int_0^x f(t)dt - x\right)=$$

$$=\lim_{x\to 0}\frac{1}{x^{n+1}}(F(x)-F(0)-x)=$$

$$=\lim_{x\to 0}\frac{F'(x)-(F(0))'-x'}{(x^{n+1})'}=$$

$$=\lim_{x\to 0}\frac{f(x)-1}{(n+1)x^n}=\frac{1}{n+1}\cdot\lim_{x\to 0}\frac{f'(x)}{nx^{n-1}}=$$

$$=\frac{1}{n(n+1)}\lim_{x\to 0}\frac{n\cdot\cos x\cdot\sin^{n-1}x\cdot e^{\sin^n x}}{x^{n-1}}=$$

$$=\frac{1}{n+1}\lim_{x\to 0}(\cos x\cdot e^{\sin^n x})\cdot\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{n-1}=$$

$$=\frac{1}{n+1}\cdot 1\cdot 1=\frac{1}{n+1}$$

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