ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c\geq 0$ then:

$$9+2\sum_{cyc}a^5\geq\sum_{cyc}a(a^2+3a+1)$$

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We will prove that:

$$2a^5-a^3-3a^2-a+3\geq 0,\forall a\geq 0$$

$$2a^5-a^3-3a^2-a+3=$$

$$=2a^5-2a^4+2a^4-2a^3+a^3-a^2-2a^2+2a-3a+3=$$

$$=2a^4(a-1)+2a^3(a-1)+a^2(a-1)-2a(a-1)-3(a-1)=$$

$$=(a-1)(2a^4+2a^3+a^2-2a-3)=$$

$$=(a-1)(2a^4-2a^3+4a^3-4a^2+5a^2-5a+3a-3)=$$

$$=(a-1)\left(2a^3(a-1)+4a^2(a-1)+5a(a-1)+3(a-1)\right)=$$

$$=(a-1)^2(2a^3+4a^2+5a+3)\geq 0$$

$$\sum_{cyc}\left(2a^5-a^3-3a^2-a+3\right)\geq 0$$

$$2\sum_{cyc}a^5+\sum_{cyc}3\geq\sum_{cyc}(a^3+3a^2+a)$$

$$9+2\sum_{cyc}a^5\geq\sum_{cyc}a(a^2+3a+1)$$

Equality holds for: $a=b=c=1$.