ROMANIAN MATHEMATICAL MAGAZINE

If $x,y,z>0$ $x+y+z=3$ then:

$$\sum_{cyc}\frac{(\sqrt[3]{x}-\frac{1}{2}\sqrt[3]{y})^2+(\frac{\sqrt{3}}{2}\sqrt[3]{y})^2}{x+y}\geq\frac{3}{2}$$

Proposed by Khaled Abd Imouti-Damascus-Syria

Solution by Cosghun Memmedov-Azerbaijan

$$\sum_{cyc}\frac{(\sqrt[3]{x}-\frac{1}{2}\sqrt[3]{y})^2+(\frac{\sqrt{3}}{2}\sqrt[3]{y})^2}{x+y}\geq\frac{3}{2}$$

$$let\ \sqrt[3]{x}=a,\sqrt[3]{y}=b,\sqrt[3]{z}=c$$

$$a^3+b^3+c^3=3,\quad \frac{a^3+b^3+c^3}{3}\geq\left(\frac{a+b+c}{3}\right)^3\quad =>a+b+c\leq 3$$

$$\sum_{cyc}\frac{(a-\frac{1}{2}b)^2+(\frac{\sqrt{3}}{2}b)^2}{a^3+b^3}$$

$$\geq\sum_{cyc}\frac{a^2-ab+b^2}{(a+b)(a^2-ab+b^2)}=\sum_{cyc}\frac{1}{a+b}\overset{Bergstrom}{\geq}\frac{9}{2(a+b+c)}\geq\frac{3}{2}$$