ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0$ then:

$$a+b+c-\sqrt[3]{abc}\ge\frac{2}{3}\left(\sqrt{c}\cdot\sqrt[4]{ab}+\sqrt{a}\cdot\sqrt[4]{bc}+\sqrt{b}\cdot\sqrt[4]{ca}\right)$$

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Solution by Daniel Sitaru-Romania

$$a+b+c-\sqrt[3]{abc}\overset{AM-GM}{\ge}a+b+c-\frac{a+b+c}{3}=\frac{2}{3}(a+b+c)=$$

$$=\frac{2}{3}\left(\frac{a+b+c+c}{4}+\frac{b+c+a+a}{4}+\frac{c+a+b+b}{4}\right)\ge$$

$$\overset{AM-GM}{\ge}\frac{2}{3}\left(\sqrt[4]{a\cdot b\cdot c\cdot c}+\sqrt[4]{b\cdot c\cdot a\cdot a}+\sqrt[4]{c\cdot a\cdot b\cdot b}\right)=$$

$$=\frac{2}{3}\left(\sqrt{c}\cdot\sqrt[4]{ab}+\sqrt{a}\cdot\sqrt[4]{bc}+\sqrt{b}\cdot\sqrt[4]{ca}\right)$$

Equality holds for: $a=b=c$.