

ROMANIAN MATHEMATICAL MAGAZINE

If $a + b + c = 3$, then :

$$a^a + b^b + c^c \geq 3$$

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$$x^x \geq x \Leftrightarrow x \cdot \ln x \geq \ln x \Leftrightarrow (x - 1) \cdot \ln x \geq 0 \rightarrow (1)$$

Case 1 $x \geq 1$ and then : $x - 1 \geq 0$ and $\ln x \geq 0 \Rightarrow (x - 1) \cdot \ln x \geq 0$
 $\Rightarrow (1)$ is true

Case 2 $x < 1$ and then : $x - 1 < 0$ and $\ln x < 0 \Rightarrow (x - 1) \cdot \ln x > 0$
 $\Rightarrow (1)$ is true

\therefore combining both cases, (1) is *always* true $\therefore x^x \geq x$ and $\therefore a^a \geq a, b^b \geq b, c^c$
 $\geq c$

$\therefore a^a + b^b + c^c \geq a + b + c = 3, "="$ iff $a = b = c = 1$ (QED)