ROMANIAN MATHEMATICAL MAGAZINE

$$If\ a,b,c>0\ and\ ab+bc+ca+abc=4$$
$$then\ prove\ that\ a+b+c+abc\ \geq 4$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$We\ need\ to\ show$$

$$a+b+c+abc\geq 4\ or, a+b+c+abc\geq ab+bc+ca+abc\ or$$

$$a+b+c\geq ab+bc+ca---\ (1)$$

$$Let\ a=\frac{2x}{y+z}, b=\frac{2y}{z+x}, c=\frac{2z}{x+y}\ , now\ from\ (1)\sum\frac{x}{y+z}\geq\sum\frac{2xy}{(y+z)(z+x)}$$

$$or, \sum x(x+y)(x+z)\geq\sum 2xy(x+y)\ \ or$$
$$\sum x^3+3xyz\geq\sum xy(x+y)\ \ (Schur's\ inequality).$$

$$Equality\ for\ x=y=z=1\ or\ a=b=c=1$$