ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0$, then prove that:

$$\frac{a^3}{bc}+\frac{b^3}{ca}+\frac{c^3}{ab}+4\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)\geq 9$$

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$$\frac{a^3}{bc}+\frac{b^3}{ca}+\frac{c^3}{ab}+4\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)=\sum_{cyc}\left(\frac{a^3}{2bc}+\frac{a^3}{2bc}+\frac{4}{a^2}\right)\overset{A-G}{\geq}3\sum_{cyc}\sqrt[3]{\frac{a^4}{b^2c^2}}$$

$$=\frac{3}{\sqrt[3]{a^2b^2c^2}}\sum_{cyc}a^2\overset{A-G}{\geq}\frac{9\sqrt[3]{a^2b^2c^2}}{\sqrt[3]{a^2b^2c^2}}.=9, \therefore \frac{a^3}{bc}+\frac{b^3}{ca}+\frac{c^3}{ab}+4\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)\geq 9$$

$$\forall\, a,b,c>0,''='' \text{ iff } a=b=c \text{ and for } a^5=8a^2 \text{ and analogs}$$

$$\Rightarrow \text{iff } a=b=c=2 \text{ (QED)}$$