ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0, (a+1)(b+1)(c+1)=8$ then:

$$a^2+b^2+c^2\geq 3$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$Let\ \ 1+a=x,\qquad 1+b=y,\qquad 1+c=z$$
$$Then\ \ xyz=8.$$
$$x+y+z\geq 3\sqrt[3]{xyz}=3\cdot 2=6$$

$$(x-1)^2+(y-1)^2+(z-1)^2\geq 3\ (to\ prove)$$

$$\frac{(x-1)^2}{1}+\frac{(y-1)^2}{1}+\frac{(z-1)^2}{1}\overset{Bergstrom}{\geq}\frac{(x+y+z-3)^2}{1+1+1}\geq\frac{(6-3)^2}{3}=3$$

Equality holds for $a=b=c=1$.