

ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0$, $ab + bc + ca = abc$ then:

$$\frac{a^4 + b^4}{ab(a^3 + b^3)} + \frac{b^4 + c^4}{bc(b^3 + c^3)} + \frac{c^4 + a^4}{ac(c^3 + a^3)} \geq 1$$

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Solution by Tapas Das-India

$$\begin{aligned} & \frac{a^4 + b^4}{ab(a^3 + b^3)} + \frac{b^4 + c^4}{bc(b^3 + c^3)} + \frac{c^4 + a^4}{ac(c^3 + a^3)} \stackrel{\text{CEBYSHEV}}{\geq} \\ & \geq \sum \frac{1}{2} \frac{(a+b)(a^3 + b^3)}{ab(a^3 + b^3)} \geq \frac{1}{2} \sum \frac{a+b}{ab} = \sum \frac{1}{a} = \frac{\sum ab}{abc} = 1 \end{aligned}$$

Equality holds for: $a = b = c$.