ROMANIAN MATHEMATICAL MAGAZINE

If $x, y, z > 0$ then:

$$\frac{x}{yz} + \frac{y}{zx} + \frac{z}{xy} + 27(x^3 + y^3 + z^3) \geq 12$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Ravi Prakash-New Delhi-India

For $t > 0, \frac{1}{t} + 27t^3 = \frac{1}{3t} + \frac{1}{3t} + \frac{1}{3t} + 27t^3 \geq 4\left(\frac{1}{3^3t^3} \cdot 27t^3\right)^{\frac{1}{4}} = 4$

Equality when $t = \frac{1}{3}$. For $x, y, z > 0$

$$\frac{1}{2}\left(\frac{x}{yz} + \frac{y}{zx}\right) + 27z^3 \geq \sqrt{\frac{x}{yz} \cdot \frac{y}{zx}} + 27z^3$$

$$\geq \frac{1}{z} + 27z^3 \geq 4 \quad (1)$$

Similarly, $\frac{1}{2}\left(\frac{x}{yz} + \frac{z}{xy}\right) + 27y^3 \geq 4 \quad (2)$

and $\frac{1}{2}\left(\frac{y}{zx} + \frac{z}{xy}\right) + 2yx^3 \geq 12 \quad (3)$

Adding (1), (2) and (3), we get the desired inequality.