ROMANIAN MATHEMATICAL MAGAZINE

If $a, b \in \mathbb{R}$ and $a^2 + b^2 \leq a + b$, then prove that :

$$a(a+1) + b(b+1) \leq 4$$

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$$a + b \geq a^2 + b^2 \geq \frac{(a+b)^2}{2} \Rightarrow 2 \geq a + b \ (\because a + b \geq a^2 + b^2 \geq 0)$$

$$\Rightarrow 4 \geq 2a + 2b \overset{a+b \geq a^2+b^2}{\geq} a + b + a^2 + b^2 \Rightarrow 4 \geq a(a+1) + b(b+1),$$

$$''='' \text{ iff } a = b = 1 \ (\text{QED})$$