

ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0$, $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} \leq 3$ then:

$$\frac{1}{(a+1)^2} + \frac{1}{(b+1)^2} + \frac{1}{(c+1)^2} \leq \frac{3}{4}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$\begin{aligned} \frac{1}{(a+1)^2} + \frac{1}{(b+1)^2} + \frac{1}{(c+1)^2} &\stackrel{AM-GM}{\leq} \sum \frac{1}{(2\sqrt{a})^2} = \\ &= \frac{1}{4} \sum \frac{1}{a} = \frac{1}{4} \sum \sqrt{\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)^2} \leq \\ &\leq \frac{1}{4} \sum \sqrt{\left(3\left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}\right)\right)} \leq \frac{1}{4} \sqrt{9} = \frac{3}{4} \left(\text{since, } \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} \leq 3\right) \end{aligned}$$