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If $a, b, c \in \mathbb{R}$ and $ab(a+b)+bc(b+c)+ca(c+a) \geq 6$,

then prove that : $a^2+b^2+c^2 \geq 3$

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$$6 \leq \sum_{cyc} bc(b+c) \overset{CBS}{\leq} \sqrt{\sum_{cyc} a^2b^2} \cdot \sqrt{\sum_{cyc} (b+c)^2}$$

$$\leq \sqrt{\frac{\left(\sum_{cyc} a^2\right)^2}{3}} \cdot \sqrt{2\sum_{cyc} a^2 + 2\sum_{cyc} ab} \leq \sqrt{\frac{\left(\sum_{cyc} a^2\right)^2}{3}} \cdot \sqrt{4\sum_{cyc} a^2} \Rightarrow \frac{4\left(\sum_{cyc} a^2\right)^3}{3} \geq 36$$

$$\Rightarrow \sum_{cyc} a^2 \geq 3 \text{ (QED)}$$