ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0$ then:

$$\frac{1}{a^4(a+b)}+\frac{1}{b^4(b+c)}+\frac{1}{c^4(c+a)} \geq \frac{3}{2}$$

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Solution by Tapas Das-India

$$\frac{1}{a^4(a+b)}+\frac{1}{b^4(b+c)}+\frac{1}{c^4(c+a)} \overset{abc=1}{=} \sum \frac{a^4b^4c^4}{a^4(a+b)} =$$

$$= \sum \frac{(b^2c^2)^2}{a+b} \geq \frac{(\sum b^2c^2)^2}{2(a+b+c)} = \frac{(\sum b^2c^2)(\sum b^2c^2)}{2(a+b+c)} \overset{AM-GM}{\geq}$$

$$\geq \frac{3(abc)^{\frac{2}{3}}abc(a+b+c)}{2(a+b+c)} = \frac{3}{2} \ (since\ abc = 1)$$

Equality holds for $a = b = c = 1$.