

ROMANIAN MATHEMATICAL MAGAZINE

If $a, b > 0$ then:

$$\frac{a+b}{\sqrt{a(4a+5b)} + \sqrt{b(4b+5a)}} \geq \frac{1}{3}$$

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Solution by Tapas Das-India

$$\begin{aligned} & \sqrt{a(4a+5b)} + \sqrt{b(4b+5a)} \stackrel{CBS}{\leq} \sqrt{2(4a^2 + 4b^2 + 10ab)} = \\ & = \sqrt{2(4(a+b)^2 + 2ab)} \stackrel{AM-GM}{\leq} \sqrt{2\left(4(a+b)^2 + \frac{(a+b)^2}{2}\right)} = 3(a+b) \\ & \frac{a+b}{\sqrt{a(4a+5b)} + \sqrt{b(4b+5a)}} \geq \frac{a+b}{3(a+b)} = \frac{1}{3} \end{aligned}$$

Equality holds for $a = b$