

ROMANIAN MATHEMATICAL MAGAZINE

If $x, y, z > 0, x + y + z = 5$ then:

$$\sqrt[7]{5x + yz} + \sqrt[7]{5y + xz} + \sqrt[7]{5z + xy} < 5$$

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Solution by Tapas Das-India

$$5x + yz = (x + y + z)x + yz = (x + y)(x + z)$$

$$\sqrt[7]{5x + yz} = \sqrt[7]{(x + y)(x + z)1 \cdot 1 \cdot 1 \cdot 1 \cdot 1} \stackrel{AM-GM}{<} <$$

$$< \frac{x + y + x + z + 5}{7} = \frac{2x + y + z + 5}{7}$$

$$\sqrt[7]{5x + yz} + \sqrt[7]{5y + xz} + \sqrt[7]{5z + xy} = \sum_{cyc} \sqrt[7]{5x + yz} <$$

$$< \sum_{cyc} \frac{2x + y + z + 5}{7} = \frac{2 \cdot 5 + 5 + 5 + 15}{7} = \frac{35}{7} = 5$$