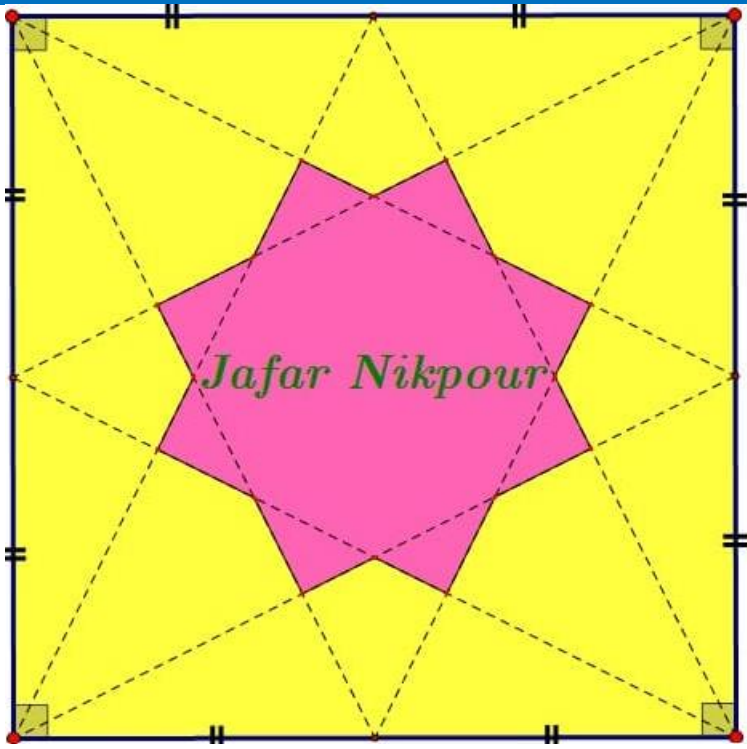
ROMANIAN MATHEMATICAL MAGAZINE



Prove that 7yellow=23pink

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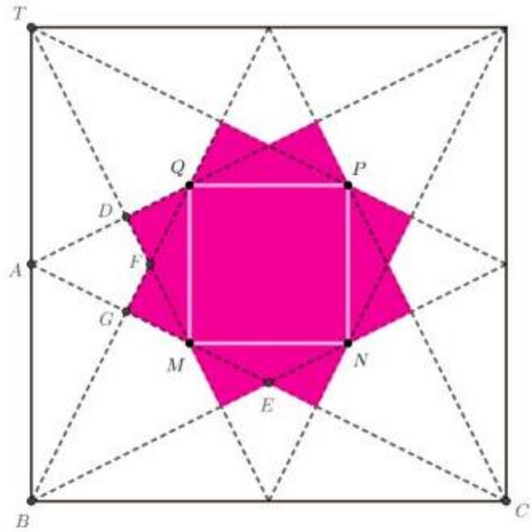
Solution by Eric Cismaru-Romania



Let x be the length of the sides of the square. It is sufficient to show that $S = \frac{7x^2}{30}$. Because $MNPQ$ is a square, $S = MN^2 + 4 \cdot (\mathcal{A}_{\Delta AQM} - \mathcal{A}_{ADFG})$. By Menelaus's Theorem in ΔTBS (where $BS = SC$), with $A - M - C$, we have:

$$\frac{TA}{BA}\cdot\frac{BC}{SC}\cdot\frac{SM}{TM}=1\Leftrightarrow\frac{AM}{CM}=\frac{1}{2}\Rightarrow AM=AQ=\frac{AC}{3}=\frac{x\sqrt{5}}{6}$$

Then, $GM=AM-AG=\frac{x\sqrt{5}}{6}-\frac{x\sqrt{5}}{10}=\frac{x\sqrt{5}}{15}$, and $GM+GQ=\frac{x\sqrt{5}}{5}\Rightarrow GQ=\frac{2x\sqrt{5}}{15}$, so we obtain that $MQ=\sqrt{GM^2+GQ^2}=\frac{\pi}{3}$

We also have $\frac{AG}{AM}=\frac{d(A;GD)}{d(A;MQ)}=\frac{3}{5}\Rightarrow d(A;GD)=\frac{x}{5}\Rightarrow d(G;MQ)=\frac{2x}{15}$, yielding to $GD=\frac{x}{5}$.

Finally, we have:

$$S=MN^2+4\cdot\left(\mathcal{A}_{\Delta AQM}-\mathcal{A}_{ADFG}\right)\Leftrightarrow S=\frac{x^2}{9}+4\cdot\left(\frac{x^2}{18}-\frac{AF\cdot GD}{2}\right),$$

Leading us to $S=\frac{x^2}{9}+4\cdot\left(\frac{x^2}{18}-\frac{x^2}{40}\right)=\frac{7x^2}{30}$, so the proof is finished.