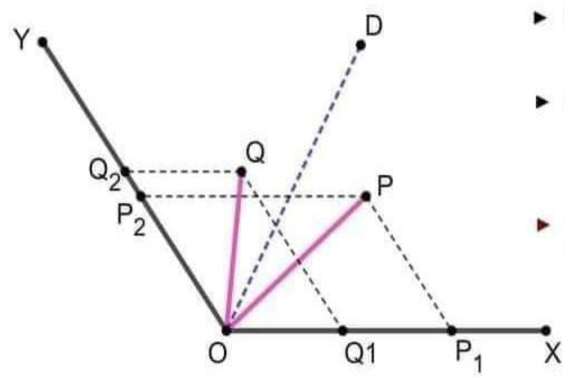
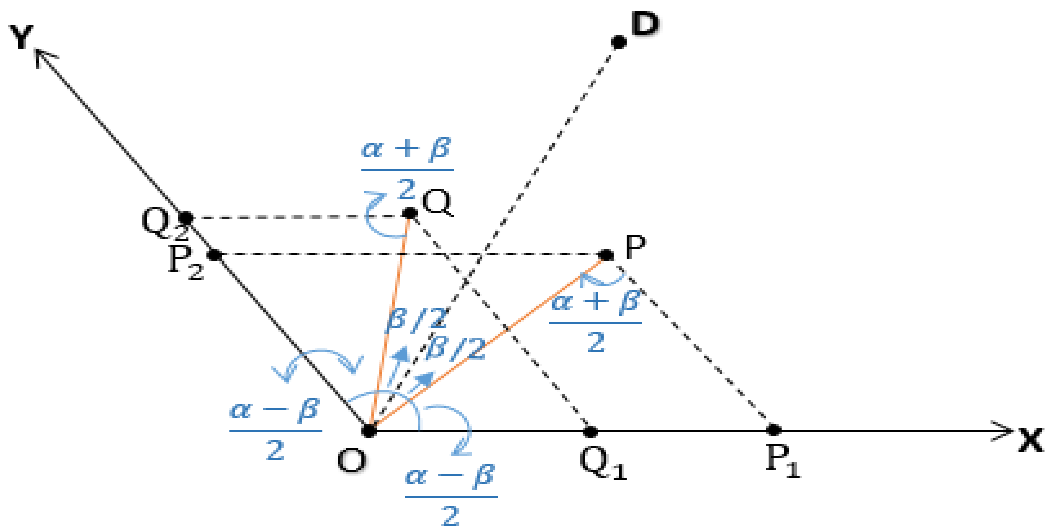
ROMANIAN MATHEMATICAL MAGAZINE



- OD bisector of $\measuredangle XOY$ and $\measuredangle POQ$. $\measuredangle XOY = \theta$, $\measuredangle POQ = \rho$
- PP_1OP_2 is #, QQ_1OQ_2 is #. $m_1 = \frac{OP_2}{OP_1}$, $m_2 = \frac{OQ_2}{OQ_1}$
- $m_1 = f_1(\theta, \rho) = ?$, $m_2 = f_2(\theta, \rho) = ?$

Proposed by Thanasis Gakopoulos – Greece

Solution by Mirsadix Muzefferov – Azerbaijan



$$m_1 = \frac{OP_2}{OP_1} = \frac{PP_1}{OP_1};\ in\ OPP_1\ rule\ sine: \frac{\sin\left(\frac{\theta-\rho}{2}\right)}{\sin\left(\frac{\theta+\rho}{2}\right)} = \frac{PP_1}{OP_1}$$

$$m_1 = \frac{OP_2}{OP_1} = \frac{PP_1}{OP_1} = \frac{\sin\left(\frac{\theta-\rho}{2}\right)}{\sin\left(\frac{\theta+\rho}{2}\right)} = \frac{\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\rho}{2}\right) - \sin\left(\frac{\rho}{2}\right)\cos\left(\frac{\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\rho}{2}\right) + \sin\left(\frac{\rho}{2}\right)\cos\left(\frac{\theta}{2}\right)} = \frac{\tan\left(\frac{\theta}{2}\right) - \tan\left(\frac{\rho}{2}\right)}{\tan\left(\frac{\rho}{2}\right) + \tan\left(\frac{\theta}{2}\right)}$$

$$m_2=\frac{OQ_2}{OQ_1}=\frac{OQ_2}{QQ_2};\ in\ OQQ_2\ rule\ sine\ :\frac{\sin\left(\frac{\theta+\rho}{2}\right)}{\sin\left(\frac{\theta-\rho}{2}\right)}=\frac{OQ_2}{QQ_2}$$

$$m_2=\frac{OQ_2}{OQ_1}=\frac{OQ_2}{QQ_2}=\frac{\sin\left(\frac{\theta+\rho}{2}\right)}{\sin\left(\frac{\theta-\rho}{2}\right)}=\frac{\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\rho}{2}\right)+\sin\left(\frac{\rho}{2}\right)\cos\left(\frac{\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\rho}{2}\right)-\sin\left(\frac{\rho}{2}\right)\cos\left(\frac{\theta}{2}\right)}=\frac{\tan\left(\frac{\theta}{2}\right)+\tan\left(\frac{\rho}{2}\right)}{\tan\left(\frac{\theta}{2}\right)-\tan\left(\frac{\rho}{2}\right)}$$