

# ROMANIAN MATHEMATICAL MAGAZINE

If in  $\triangle ABC$ ,  $a \neq b \neq c \neq a$  then:

$$\frac{1}{m_a^2 - m_b^2} \left( \frac{1}{h_a^2} - \frac{1}{h_b^2} \right)^2 + \frac{1}{m_b^2 - m_c^2} \left( \frac{1}{h_b^2} - \frac{1}{h_c^2} \right)^2 + \frac{1}{m_c^2 - m_a^2} \left( \frac{1}{h_c^2} - \frac{1}{h_a^2} \right)^2 = 0$$

Proposed by Dang Ngoc Minh-Vietnam

Solution by Tapas Das-India

$$m_a^2 - m_b^2 = \frac{2b^2 + 2c^2 - a^2}{4} - \frac{2a^2 + 2c^2 - b^2}{4} = \frac{3}{4} (b^2 - a^2)$$

$$\left( \frac{1}{h_a^2} - \frac{1}{h_b^2} \right)^2 = \left( \frac{a^2}{4F^2} - \frac{b^2}{4F^2} \right)^2 = \frac{1}{16F^2} (a^2 - b^2)^2$$

$$\frac{1}{m_a^2 - m_b^2} \left( \frac{1}{h_a^2} - \frac{1}{h_b^2} \right)^2 = \frac{1}{16F^2} (a^2 - b^2)^2 \cdot \frac{4}{3} \frac{1}{(b^2 - a^2)} =$$

$$= -\frac{1}{12F^2} (a^2 - b^2) = \frac{1}{12F^2} (b^2 - a^2)$$

$$\frac{1}{m_a^2 - m_b^2} \left( \frac{1}{h_a^2} - \frac{1}{h_b^2} \right)^2 + \frac{1}{m_b^2 - m_c^2} \left( \frac{1}{h_b^2} - \frac{1}{h_c^2} \right)^2 + \frac{1}{m_c^2 - m_a^2} \left( \frac{1}{h_c^2} - \frac{1}{h_a^2} \right)^2 =$$

$$= \sum \frac{1}{m_a^2 - m_b^2} \left( \frac{1}{h_a^2} - \frac{1}{h_b^2} \right)^2 = \sum \frac{1}{12F^2} (b^2 - a^2) =$$

$$= \frac{1}{12F^2} (b^2 - a^2 + c^2 - b^2 + a^2 - c^2) = 0$$