ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sum\sqrt{m_a^2+rr_a}=\frac{1}{2}\sum(r_c+r)\sqrt{\frac{r_a+r_b}{r_c-r}}=\sum(a+b)cosC=2s$$

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$$1)\ \sum\sqrt{m_a^2+rr_a}$$

$$m_a^2+rr_a=\frac{1}{4}((2(b^2+c^2)-a^2)+(a^2-(b-c)^2))=$$

$$=\frac{1}{4}(2b^2+2c^2-a^2+a^2-b^2-c^2+2bc)=\frac{1}{4}(b^2+c^2+2bc)=\frac{1}{4}(b+c)^2$$

$$Here\ \boxed{a^2=(b-c)^2+4rr_a}\ (true)$$

$$\sum\sqrt{m_a^2+rr_a}=\sum\frac{1}{2}(b+c)=2s\quad(1)$$

$$2)\ \frac{1}{2}\sum(r_c+r)\sqrt{\frac{r_a+r_b}{r_c-r}}$$

$$(r_c+r)=stan\frac{C}{2}+(s-c)tan\frac{C}{2}=(2s-c)tan\frac{C}{2}=(a+b)tan\frac{C}{2}$$

$$\frac{r_a+r_b}{r_c-r}=\frac{4R+r-r_c}{r_c-r}=\frac{4R}{r_c-r}-1=\frac{4R}{stan\frac{C}{2}-(s-c)tan\frac{C}{2}}-1=\frac{4R}{ctan\frac{C}{2}}-1=$$

$$\frac{4R}{4Rsin\frac{C}{2}cos\frac{C}{2}tan\frac{C}{2}}-1=\frac{1}{sin^2\frac{C}{2}}-1=ctg^2\frac{C}{2}\quad Here\ \boxed{r_a+r_b+r_c=4R+r}\ (true)$$

$$(r_c+r)\sqrt{\frac{r_a+r_b}{r_c-r}}=(a+b)tan\frac{C}{2}.cot\frac{C}{2}=(a+b)$$

$$\frac{1}{2}\sum(r_c+r)\sqrt{\frac{r_a+r_b}{r_c-r}}=2s\quad(2)$$

$$3)\ \sum(a+b)cosC$$

$$\sum(a+b)cosC=(a+b)cosC+(b+c)cosA+(a+c)cosB=$$

$$=(bcosC+ccosB)+(ccosA+acosC)+(acosB+bcosA)=a+b+c=2s$$

$$Here\ \boxed{\begin{array}{l} a=b.cosC+c.cosB \\ b=c.cosA+a.cosC \\ c=a.cosB+b.cosA \end{array}}\ (true)$$

$$\sum(a+b)cosC=2s\ \ (3)$$

The expression from (1), (2) *and* (3) *has been proved.*