ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\prod_{cyc}\frac{\frac{r_a}{r_b}+1}{\frac{h_a}{h_b}+1}=\prod_{cyc}\frac{r_a-r}{h_a-r}$$

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Solution by Tapas Das-India

$$\frac{r_a-r}{h_a-r}=\frac{\frac{F}{s-a}-r}{\frac{2F}{a}-r}=\frac{\frac{r(s-s+a)}{s-a}}{\frac{r(2s-a)}{a}}=\frac{a^2}{(s-a)(2s-a)}=\frac{a^2}{(s-a)(b+c)}$$

$$\prod_{cyc}\frac{r_a-r}{h_a-r}=\prod_{cyc}\frac{a^2}{(s-a)(b+c)}=$$

$$=\frac{a^2b^2c^2}{(s-a)(s-b)(s-c)(a+b)(b+c)(c+a)}\ (A)$$

$$\frac{\frac{r_a}{r_b}+1}{\frac{h_a}{h_b}+1}=\frac{\frac{s-b}{s-a}+1}{\frac{b}{a}+1}=\frac{a}{a+b}\frac{2s-b-a}{s-a}=\frac{ac}{(s-a)(a+b)}$$

$$\prod_{cyc}\frac{\frac{r_a}{r_b}+1}{\frac{h_a}{h_b}+1}=\prod_{cyc}\frac{ac}{(s-a)(a+b)}=$$

$$=\frac{a^2b^2c^2}{(s-a)(s-b)(s-c)(a+b)(b+c)(c+a)}\ (B)$$

$$From\ (A)\ and\ (B) we\ get\ \prod\frac{\left(\frac{r_a}{r_b}+1\right)}{\frac{h_a}{h_b}+1}=\prod\frac{r_a-r}{h_a-r}$$