ROMANIAN MATHEMATICAL MAGAZINE

In $\Delta ABC, G$ −centroid, X, Y, Z −circumradies of $\Delta BCG, \Delta CAG, \Delta ABG$.

Prove that:

$$\frac{\sin A}{X\,m_a}=\frac{\sin B}{Ym_b}=\frac{\sin C}{Zm_c}$$

Proposed by George Apostolopoulos-Greece

Solution by Tapas Das-India

$$BG=\frac{2}{3}m_b, CG=\frac{2}{3}m_c, AG=\frac{2}{3}m_a\ ,$$

$$area\ of\ the\ \ \Delta\ ABC=F\ then\ [\ BGC]=[CGA]=[AGB]=\frac{F}{3},$$

$$X=\frac{BG.CG.BC}{4[BGC]}=\frac{\frac{2}{3}m_b\frac{2}{3}m_c.a}{\frac{4F}{3}}=\frac{am_bm_c}{3F}, similarly\ Y=\frac{bm_cm_a}{3F}\ and\ Z=\frac{cm_am_b}{3F}$$

$$\frac{\sin A}{X\,m_a}=\frac{a}{2R}.\frac{3F}{am_am_bm_c}=\frac{3F}{2R}.\frac{1}{m_am_bm_c}$$

$$\frac{\sin B}{Ym_b}=\frac{b}{2R}.\frac{3F}{bm_am_bm_c}=\frac{3F}{2R}.\frac{1}{m_am_bm_c}$$

$$\frac{\sin C}{Zm_c}=\frac{c}{2R}.\frac{3F}{cm_am_bm_c}=\frac{3F}{2R}.\frac{1}{m_am_bm_c}$$

$$\frac{\sin A}{X\,m_a}=\frac{\sin B}{Ym_b}=\frac{\sin C}{Zm_c}$$