ROMANIAN MATHEMATICAL MAGAZINE

$ABC\ , A'B'C'$ *and* $A"B"C"$ *are 3 similar triangles.*

If the sides $BC, B'C'$ *and* $B"C"$ *are sides in a right triangle then the area of the biggest triangle is the sum of the areas of the other two triangle.*

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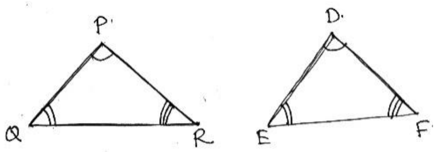

Theorem: *If* ΔPQR *and* ΔDEF *are similar* $(\angle P = \angle D, \angle Q = \angle E, \angle R = \angle F)$ *then*

$$\frac{|PQR]}{[DEF]} = \frac{QR^2}{EF^2} = \frac{PR^2}{DF^2} = \frac{PQ^2}{DE^2}$$

Proof: $\Delta PQR \sim \Delta DEF$ *and*

$(\angle P = \angle D, \angle Q = \angle E, \angle R = \angle F)$ *so corresponding sides are proportional*

$$\frac{QR}{EF} = \frac{PR}{DF} = \frac{PQ}{DE}\ (1)$$

$$Now\ \frac{|PQR]}{[DEF]} = \frac{\frac{1}{2}PQ.PR\sin P}{\frac{1}{2}DE.DF.\sin D}(since\ \angle P = \angle D) = \frac{PQ}{DE}.\frac{PR}{DF} \overset{(1)}{=} \frac{QR}{EF}.\frac{QR}{EF} = \frac{QR^2}{EF^2}$$

$$Similarly \frac{|PQR]}{[DEF]} = \frac{PR^2}{DF^2}\ and\ \frac{|PQR]}{[DEF]} = \frac{PQ^2}{DE^2}\ or$$

$$\frac{|PQR]}{[DEF]} = \frac{QR^2}{EF^2} = \frac{PR^2}{DF^2} = \frac{PQ^2}{DE^2}(proof\ complete)$$

According to the given problem $\Delta ABC, \Delta\ A'B'C'$ *and* $\Delta A"B"C"$ *are similar ,*

let side $BC > BC\ , BC > B'C'$ *and according to the question*

they form right angle triangle, so

$B"C"^2 = BC^2 + B'C'^2\ \ (2)$ *and we need to show* $[ABC] + [A'B'C'] = [A"B"C"]$

now using the above theorem we have

$$\frac{[ABC]}{BC^2} = \frac{[A'B'C']}{B'C'^2} = \frac{[A"B"C"]}{B"C"^2} = K(say).$$

$$[ABC] = K.BC^2, [A'B'C'] = K.B'C'^2, [A"B"C"] = K.B"C"^2$$

$$[ABC] + [A'B'C'] = K.BC^2 + K.B'C'^2 = K(BC^2 + B'C'^2) \overset{(2)}{=} KB"C"^2 = [A"B"C"]$$

$$or\ [ABC] + [A'B'C'] = [A"B"C"](proof\ complete)$$