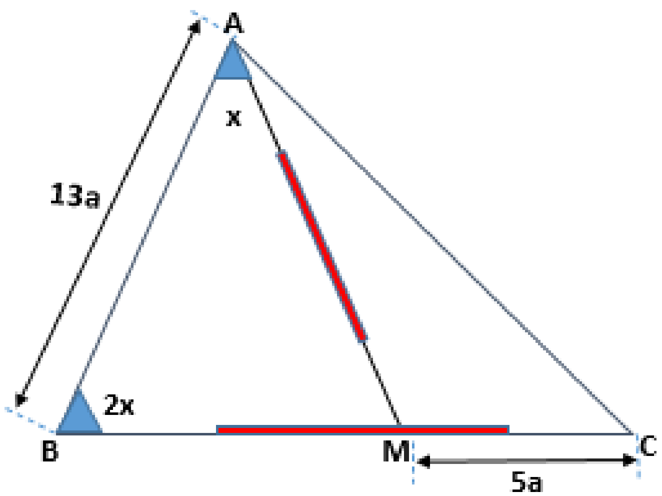
ROMANIAN MATHEMATICAL MAGAZINE

In the triangle ABC , the point M (on side BC) is such that:

$$MA = BC\ ,MC = \left(\frac{5}{13}\right)\cdot AB\ ,M\hat{B}A = 2\cdot M\hat{A}B = 2x$$

Under these conditions , determine the value of x.



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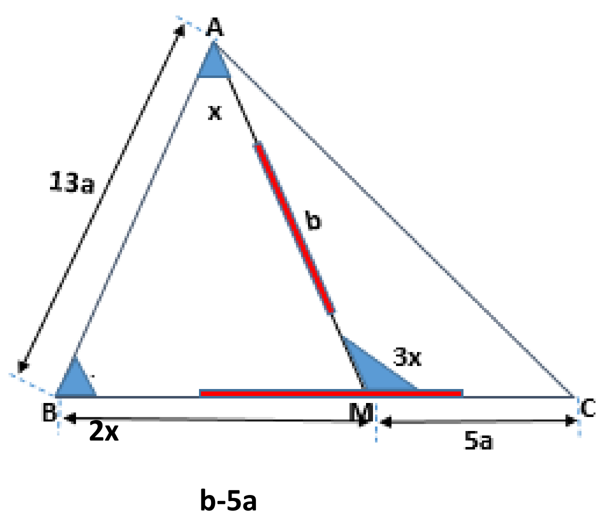
Solution by Mirsadix Muzefferov-Azerbaijan



Let AM=b.Then BM=b-5a. In ΔABM rule sine

$$\frac{\sin B\hat{M}A}{AB} = \frac{\sin A\hat{B}M}{AM} = \frac{\sin B\hat{A}M}{BM} \Rightarrow \frac{\sin 3x}{13a} = \frac{\sin 2x}{b} = \frac{\sin x}{b-5a} \Rightarrow$$

$$\Rightarrow \frac{a}{b} = \frac{\sin 3x}{13 \sin 2x} \quad (1)$$

$$\frac{\sin 2x}{\sin x} = \frac{b}{b - 5a} \Rightarrow \frac{a}{b} = \frac{\sin 2x - \sin x}{5 \sin 2x} \quad (2)$$

From (1) and (2) we obtain

$$\frac{\sin 2x - \sin x}{5 \sin 2x} = \frac{\sin 3x}{13 \sin 2x} \Rightarrow 5 \sin 3x = 13 \sin 2x - 13 \sin x \Rightarrow$$

$$\Rightarrow 5 \cdot (3 \sin x - 4 sin^3 x) = 26 \cdot \sin x \cos x - 13 \sin x \Rightarrow 15 - 20 sin^2 x =$$

$$= 26 \cos x - 13 \Rightarrow 10 cos^2 x - 13 \cos x + 4 = 0$$

Let $\cos x = t$

$$10t^2 - 13t + 4 = 0 \Rightarrow t_1 = \frac{1}{2}, t_2 = \frac{4}{5}$$

$$\cos x = \frac{1}{2} \Rightarrow x = 60^\circ$$

it does not satisfy the condition of the problem

$$\cos x = \frac{4}{5} \Rightarrow x = \cos^{-1}\left(\frac{4}{5}\right) \approx 37^\circ$$

Answer: $x = \cos^{-1}\left(\frac{4}{5}\right) \approx 37^\circ$