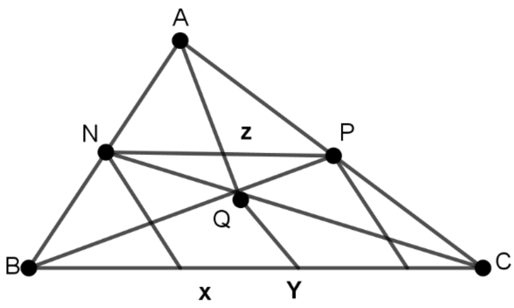
ROMANIAN MATHEMATICAL MAGAZINE



If: $NX \parallel AQ \parallel PY \parallel \Rightarrow$ Prove that:

$$\frac{XN}{NZ} \cdot \frac{ZP}{PY} = 1$$

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ΔABE and ΔBNX (They are similar)

Then: $\frac{XN}{AE} = \frac{BN}{BA}$ (1)

Also, ΔAEC and ΔPYC (are similar)

Then: $\frac{AE}{PY} = \frac{AC}{PC}$ (2)

Multiply (1) and (2) side by side:

$$\frac{XN}{AE} \cdot \frac{AE}{PY} = \frac{BN}{BA} \cdot \frac{AC}{PC} \Rightarrow \frac{XN}{PY} = \frac{BN}{BA} \cdot \frac{AC}{PC} \quad (3)$$

On the other hand, according to Tanasis Gakopoulos theorem, in ΔABC ...

$$\frac{NZ}{ZP} = \frac{BN}{AB} : \frac{CP}{AC} \text{ or } \frac{ZP}{NZ} = \frac{AB}{BN} \cdot \frac{CP}{AC} \quad (4)$$

Multiply (3) and (4) side by side:

$$\frac{XN}{PY} \cdot \frac{NZ}{ZP} = \left(\frac{BN}{BA} \cdot \frac{AC}{PC}\right) \cdot \left(\frac{AB}{BN} \cdot \frac{CP}{AC}\right) = 1$$