

# ROMANIAN MATHEMATICAL MAGAZINE

If in  $\Delta ABC$ ,  $A:B:C = 1:3:6$  then find :  $\frac{s}{r}$

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**Solution by proposer**

$$A:B:C = 1:3:6 \Rightarrow B = 3A, C = 6A$$

$$A + B + C = 180^\circ \Rightarrow A + 3A + 6A = 180^\circ \Rightarrow 10A = 180^\circ,$$

$$A = 18^\circ, B = 54^\circ, C = 108^\circ$$

$$F = \frac{1}{2} bcsinA \Rightarrow rs = \frac{1}{2} bcsinA \Rightarrow r = \frac{bcsinA}{2s}$$

$$\frac{r}{s} = \frac{bcsinA}{2s^2} = \frac{2RsinB \cdot 2RsinC \cdot sinA}{2 \cdot \left(\frac{a+b+c}{2}\right)^2} =$$

$$= \frac{2R^2 sinAsinBsinC}{(RsinA + RsinB + RsinC)^2} = \frac{2sinAsinBsinC}{(sinA + sinB + sinC)^2}$$

$$\frac{s}{r} = \frac{(sinA + sinB + sinC)^2}{2sinAsinBsinC} = \frac{(sin18^\circ + sin54^\circ + sin108^\circ)^2}{2sin18^\circ sin54^\circ sin108^\circ}$$

$$sin18^\circ = \frac{\sqrt{5}-1}{4}, \quad sin54^\circ = \frac{\sqrt{5}+1}{4}, \quad sin108^\circ = \frac{\sqrt{5}+\sqrt{5}}{\sqrt{8}}$$

Then:

$$\frac{s}{r} = \frac{15 + \sqrt{5} + 2\sqrt{50 + 10\sqrt{5}}}{\sqrt{10 + 2\sqrt{5}}}$$