

ROMANIAN MATHEMATICAL MAGAZINE

If in $\triangle ABC$, $A : B : C = 1 : 3 : 6$ then find : $\frac{s}{r}$

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Solution by proposer

$$A : B : C = 1 : 3 : 6 \Rightarrow B = 3A, C = 6A$$

$$A + B + C = 180^\circ \Rightarrow A + 3A + 6A = 180^\circ \Rightarrow 10A = 180^\circ,$$

$$A = 18^\circ, B = 54^\circ, C = 108^\circ$$

$$F = \frac{1}{2}bc\sin A \Rightarrow rs = \frac{1}{2}bc\sin A \Rightarrow r = \frac{bc\sin A}{2s}$$

$$\frac{r}{s} = \frac{bc\sin A}{2s^2} = \frac{2R\sin B \cdot 2R\sin C \cdot \sin A}{2 \cdot \left(\frac{a+b+c}{2}\right)^2} =$$

$$= \frac{2R^2\sin A\sin B\sin C}{(R\sin A + R\sin B + R\sin C)^2} = \frac{2\sin A\sin B\sin C}{(\sin A + \sin B + \sin C)^2}$$

$$\frac{s}{r} = \frac{(\sin A + \sin B + \sin C)^2}{2\sin A\sin B\sin C} = \frac{(\sin 18^\circ + \sin 54^\circ + \sin 108^\circ)^2}{2\sin 18^\circ\sin 54^\circ\sin 108^\circ}$$

$$\sin 18^\circ = \frac{\sqrt{5}-1}{4}, \quad \sin 54^\circ = \frac{\sqrt{5}+1}{4}, \quad \sin 108^\circ = \frac{\sqrt{5+\sqrt{5}}}{\sqrt{8}}$$

Then:

$$\frac{s}{r} = \frac{15 + \sqrt{5} + 2\sqrt{50 + 10\sqrt{5}}}{\sqrt{10 + 2\sqrt{5}}}$$