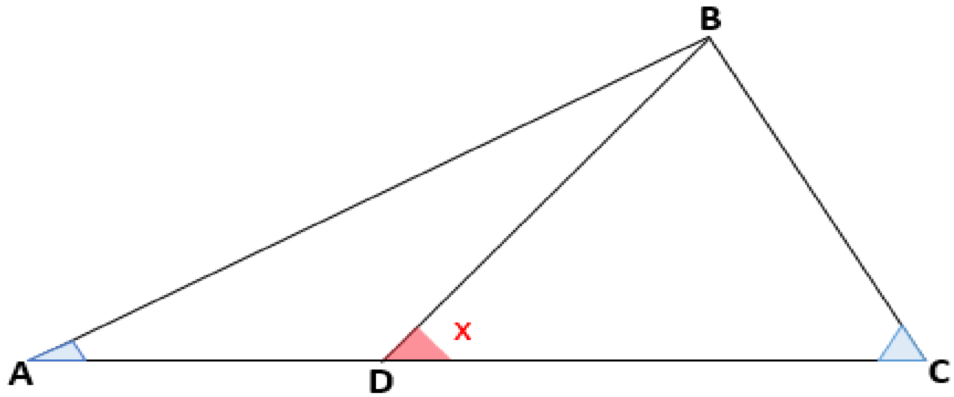
ROMANIAN MATHEMATICAL MAGAZINE

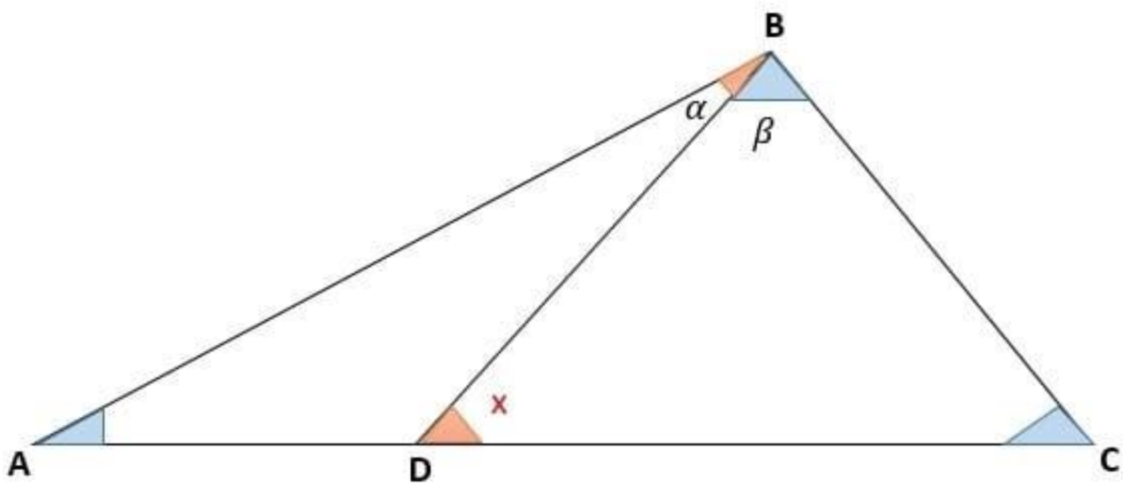


If $\frac{AD}{DB} = k$ then:

$$\tan x = \frac{(k+1)\tan A \cdot \tan B}{\tan B - k \cdot \tan A}$$

Proposed by Thanasis Gakopoulos-Greece

Solution by Mirsadix Muzefferov-Azerbaijan



$$In\ \Delta ADC\ rule\ sines\ \frac{AD}{\sin\alpha} = \frac{DC}{\sin A}$$

$$In\ \Delta DCB\ also, rule\ sines:\ \frac{DB}{\sin\beta} = \frac{DC}{\sin B}$$

$$\begin{cases}\dfrac{AD}{\sin\alpha}=\dfrac{DC}{\sin A}\\ \dfrac{DB}{\sin\beta}=\dfrac{DC}{\sin B}\end{cases}\Rightarrow\frac{AD}{\sin\alpha}:\frac{DB}{\sin\beta}=\frac{DC}{\sin A}:\frac{DC}{\sin B}\Rightarrow\frac{AD}{DB}\cdot\frac{\sin\beta}{\sin\alpha}=\frac{\sin B}{\sin A}\Rightarrow k\cdot\frac{\sin\beta}{\sin\alpha}=$$

$$=\frac{\sin B}{\sin A}\Rightarrow\beta=180^\circ-(B+x)\,;\ \alpha=x-A$$

$$k\cdot\frac{\sin(B+x)}{\sin(x-A)}=\frac{\sin B}{\sin A}\Rightarrow k\cdot\frac{\sin B\cos x+\sin x\cos B}{\sin x\cos A-\sin A\cos x}=\frac{\sin B}{\sin A}\Rightarrow$$

$$\Rightarrow k\cdot\frac{\tan B+\tan x}{\tan B\dfrac{\cos A}{\cos B}-\dfrac{\sin A}{\cos B}}=\frac{\sin B}{\sin A}\Rightarrow k\cdot(\tan B+\tan x)=\tan x\cdot\frac{\tan B}{\tan A}-\tan B$$

$$(k+1)\tan B=\tan x\left(\frac{\tan B}{\tan A}-k\right)\Rightarrow$$

$$\tan x=\frac{(k+1)\tan B}{\dfrac{\tan B}{\tan A}-k}=\frac{(k+1)\tan B\cdot\tan A}{\tan B-k\cdot\tan A}$$