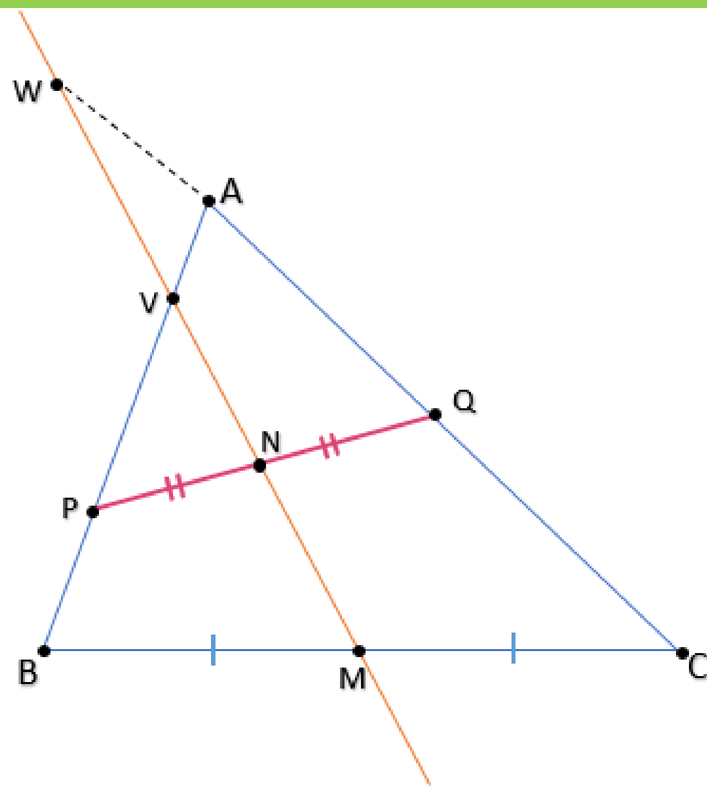
ROMANIAN MATHEMATICAL MAGAZINE



Prove that:

$$\frac{VP - AV}{AQ} = \frac{PB}{QC}\left(= \frac{AV}{AW} \text{ by Dao Thanh Oai}\right)$$

Proposed by Thanasis Gakopoulos-Greece

Solution by Mirsadix Muzefferov-Azerbaijan

$$\text{In } \Delta APQ \text{ by theorem Menelaus}:$$

$$\frac{WA}{WQ} \cdot \frac{QN}{NP} \cdot \frac{PV}{VA} = 1 \Rightarrow \frac{PV}{AV} = \frac{WA}{WQ} \Rightarrow \frac{PV}{AV} - 1 = \frac{WQ}{WA} - 1 \Rightarrow \frac{PV - AV}{AV} = \frac{WQ - WA}{WA} \Rightarrow$$

$$\Rightarrow \frac{PV - AV}{AV} = \frac{AQ}{WA} \Rightarrow \frac{PV - AV}{AQ} = \frac{AV}{AW}$$

$$\text{Now let us prove that}:$$

$$\frac{PB}{QC} = \frac{AV}{AW}$$

$$\textit{For this also by Menelaus theorem in the triangle } ABC$$

$$\frac{WA}{WC}\cdot\frac{CM}{MB}\cdot\frac{BV}{VA}=1\Rightarrow\frac{WC}{WA}=\frac{BV}{AV}\Rightarrow\frac{AV}{AW}=\frac{BV}{WC}=\frac{BP+PV}{WA+AC}\Rightarrow\frac{AV}{WA}=\frac{BP+PV}{WA+AC}\Rightarrow$$

$$\Rightarrow\frac{PB+PV}{AV}=\frac{WA+AC}{WA}\Rightarrow\frac{BP}{VA}+\frac{PV}{VA}=\frac{AC}{WA}+1\Rightarrow\frac{BP}{VA}=\frac{AC}{WA}+1-\frac{WQ}{WA}\Rightarrow$$

$$\Rightarrow\frac{BP}{VA}=\frac{AC-WQ}{WA}+1\Rightarrow\frac{AC+WA-WQ}{WA}$$

$$\frac{BP}{VA}=\frac{AQ+QC+WA-WA-AQ}{WA}=\frac{QC}{WA}\Rightarrow\frac{PB}{QC}=\frac{AV}{AW}\ (Qed)$$