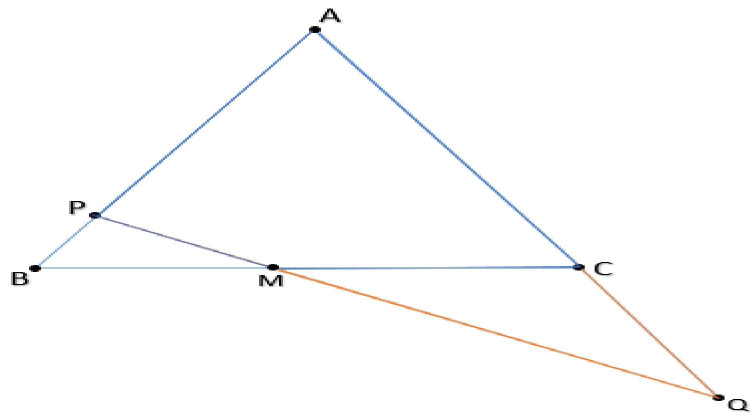
ROMANIAN MATHEMATICAL MAGAZINE



$$AB = BC = CA\,, BM = MC\,, [APMC] = S_1\,, [CMQ] = S_2\,, \frac{AP}{PB} = x > 1\,, \frac{S_1}{S_2} = y$$

$$Calculate\ y = f(x).\ If\ [APMC] = [CQM]\ then\ find\ \frac{AP}{PB}.$$

Proposed by Thanasis Gakopoulos-Greece

Solution by Mirsadix Muzefferov-Azerbaijan

$$\frac{S_1 + S_2}{S_2} = \frac{\frac{1}{2}AQ \cdot PQ \cdot \sin Q}{\frac{1}{2}CQ \cdot QM \cdot \sin Q} = \frac{AQ \cdot PQ}{CQ \cdot QM} \quad (*)$$

$$So\ let's\ find\ \frac{AQ}{CQ} = ?\ \ \frac{PQ}{QM} = ?.\ Let's\ use\ Menelaus\ theorem\ for\ this:$$

$$\frac{QC \cdot AP \cdot BM}{QA \cdot BP \cdot MC} = 1 \Rightarrow \frac{QC}{QA} = \frac{1}{x}\ (1) \Rightarrow \frac{AC}{CQ} = x - 1, \qquad \frac{AP}{BP} = x \Rightarrow \frac{BP}{BA} = \frac{1}{x+1}$$

$$Also\,, Menelaus\ theorem:$$

$$\frac{BP \cdot AC \cdot MQ}{BA \cdot CQ \cdot PM} = 1 \Rightarrow \frac{MQ}{PM} = \frac{x+1}{x-1} \Rightarrow \frac{PQ}{MQ} = \frac{2x}{x+1} \quad (2)$$

$$Let's\ use\ (1)\ and\ (2)\ in\ (*)$$

$$Then\ \frac{S_1 + S_2}{S_2} = \frac{2x^2}{x+1} \Rightarrow y = \frac{S_1}{S_2} = \frac{2x^2 - x - 1}{x+1}\ ;\ if\ S_1 = S_2 \Rightarrow y = 1$$

$$2x^2 - x - 1 = x + 1 \Rightarrow x^2 - x - 1 = 0$$

$$x = \frac{\sqrt{5} + 1}{2} = \varphi$$