ROMANIAN MATHEMATICAL MAGAZINE

Let a, b, c be sides in the right triangle $ABC, A = 90°$. Prove that:

$$c^{c^2}.b^{b^2} \geq \left(\frac{b+c}{2}\right)^{a^2}$$

Proposed by Dang Ngoc Minh-Vietnam

Solution by Tapas Das-India

$$\Delta ABC \text{ right triangle at } A,\quad b^2+c^2=a^2 \text{ and } \frac{b^2+c^2}{2} \overset{CBS}{\geq} \frac{(b+c)^2}{4} \text{ or}$$

$$(b^2+c^2) \geq \frac{(b+c)^2}{2} \quad (1)$$

Let us consider c with associated weight c^2 and b with associated weight b^2

$$G.M \geq H.M \text{ or } \left(c^{c^2}.b^{b^2}\right)^{\frac{1}{c^2+b^2}} \geq \frac{c^2+b^2}{\frac{c^2}{c}+\frac{b^2}{b}} = \frac{c^2+b^2}{c+b} \overset{(1)}{\geq} \frac{(c+b)^2}{2(c+b)} = \frac{c+b}{2}$$

$$\text{or } c^{c^2}.b^{b^2} \geq \left(\frac{c+b}{2}\right)^{c^2+b^2} = \left(\frac{b+c}{2}\right)^{a^2} \quad (\text{as } b^2+c^2=a^2)$$

$$\text{Equality holds for: } A=\frac{\pi}{2}, B=C=\frac{\pi}{4}$$