ROMANIAN MATHEMATICAL MAGAZINE

In $\Delta ABC$ the following relationship holds:

$$(h_a-2r)^{\frac{1}{h_a-2r}}.(h_b-2r)^{\frac{1}{h_b-2r}}.(h_c-2r)^{\frac{1}{h_c-2r}}\le\left(\frac{3R^2}{2R-r}\right)^{\frac{2R-r}{r^2}}$$

*Proposed by Dang Ngoc Minh-Vietnam*

*Solution by Tapas Das-India*

$$\sum\frac{a}{s-a}=\frac{\sum a(s-b)(s-c)}{(s-a)(s-b)(s-c)}=\frac{\sum a(s^2-s(b+c)+bc)}{(s-a)(s-b)(s-c)}=$$

$$=\frac{s^2(a+b+c)-2s(ab+bc+ca)+3abc}{(s-a)(s-b)(s-c)}=\frac{2s^3-2s(s^2+r^2+4Rr)+12Rrs}{sr^2}=$$

$$=\frac{2s^2-2s^2-2r^2-8Rr+12Rr}{r^2}=\frac{4R-2r}{r}\quad(1)$$

$$\sum\frac{1}{(h_a-2r)}=\sum\frac{1}{\left(\frac{2rs}{a}-2r\right)}=\frac{1}{2r}\sum\frac{a}{s-a}\overset{(1)}{=}\frac{1}{2r}\,\frac{4R-2r}{r}=\frac{2R-r}{r^2}\quad(2)$$

*Let us consider* $(h_a-2r)$ *with associated weight* $\frac{1}{(h_a-2r)}$,

$(h_b-2r)$ *with associated weight* $\frac{1}{(h_b-2r)}$ *and* $(h_c-2r)$ *with* $\frac{1}{(h_c-2r)}$.

$$By\ G.M\le A.M\ or\ \left((h_a-2r)^{\frac{1}{h_a-2r}}.(h_b-2r)^{\frac{1}{h_b-2r}}.(h_c-2r)^{\frac{1}{h_c-2r}}\right)^{\frac{1}{\sum\frac{1}{(h_a-2r)}}}\le$$

$$\le\frac{(h_a-2r).\frac{1}{(h_a-2r)}+(h_b-2r).\frac{1}{(h_b-2r)}+(h_c-2r).\frac{1}{(h_c-2r)}}{\sum\frac{1}{(h_a-2r)}}\overset{(2)}{=}\frac{3}{\frac{2R-r}{r^2}}=\frac{3r^2}{2R-r}$$

$$\left((h_a-2r)^{\frac{1}{h_a-2r}}.(h_b-2r)^{\frac{1}{h_b-2r}}.(h_c-2r)^{\frac{1}{h_c-2r}}\right)^{\frac{1}{\sum\frac{1}{(h_a-2r)}}}\le\frac{3r^2}{2R-r}$$

$$(h_a-2r)^{\frac{1}{h_a-2r}}.(h_b-2r)^{\frac{1}{h_b-2r}}.(h_c-2r)^{\frac{1}{h_c-2r}}\le\left(\frac{3r^2}{2R-r}\right)^{\sum\frac{1}{(h_a-2r)}}\overset{(2)}{=}\left(\frac{3R^2}{2R-r}\right)^{\frac{2R-r}{r^2}}$$

*Equality* $a=b=c$