ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$(2b+2c-a)a \geq 4\sqrt{3}F$$

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Solution by Tapas Das-India

$$(2b+2c-a)a = 2ab+2ac-a^2 = 2(ab+bc+ca)-a^2-2bc \overset{AM-GM}{\geq}$$

$$\geq 2(ab+bc+ca)-a^2-(b^2+c^2) = 2(ab+bc+ca)-(a^2+b^2+c^2) =$$

$$= 2(s^2+r^2+4Rr)-2(s^2-r^2-4Rr) = 4r(4R+r) \overset{Doucet}{\geq} 4r\sqrt{3}s = 4\sqrt{3}F$$

$$Equality\ holds\ for\ a=b=c$$