ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sqrt[4]{\sum \frac{Ra^8}{r_b + r_c}} \geq \frac{4F}{\sqrt{3}}$$

Proposed by Dang Ngoc Minh-Vietnam

Solution by Tapas Das-India

$$\sum (r_b + r_c) = 2\sum r_a = 2(4R + r) \overset{Euler}{\leq} 2\left(4R + \frac{R}{2}\right) = 9R \ (1)$$

$$\left(\sum \frac{a^8}{r_b + r_c}\right)\left(\sum (r_b + r_c)\right)(1 + 1 + 1)^6 \overset{Holder}{\geq} (a + b + c)^8 = (2s)^8 = 2^8 s^8$$

$$\left(\sum \frac{a^8}{r_b + r_c}\right) \geq \frac{2^8 s^8}{(\sum (r_b + r_c))3^6} \overset{(1)}{\geq} \frac{2^8 s^8}{3^6 . 9R} = \left(\frac{2}{3}\right)^8 \frac{s^8}{R} \ (2)$$

$$\sqrt[4]{\sum \frac{Ra^8}{r_b + r_c}} \overset{(2)}{\geq} \sqrt[4]{R . \left(\frac{2}{3}\right)^8 \frac{s^8}{R}} = \frac{2^2 s^2}{3^2} = \frac{4s}{9} . s \overset{Mitrinovic}{\geq} \frac{4s}{9} . 3\sqrt{3}r = \frac{4rs}{\sqrt{3}} = \frac{4F}{\sqrt{3}}$$

$$Equality\ holds\ for\ a = b = c$$