ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{1}{(a+b)(m_a+m_b-m_c)^2}+\frac{1}{(b+c)(m_b+m_c-m_a)^2}+\frac{1}{(c+a)(m_c+m_a-m_b)^2}\ge\frac{2\sqrt{3}}{9R^3}$$

Proposed by Daniel Sitaru-Romania

Solution by Tapas Das-India

$$\frac{1}{(a+b)(m_a+m_b-m_c)^2}+\frac{1}{(b+c)(m_b+m_c-m_a)^2}+\frac{1}{(c+a)(m_c+m_a-m_b)^2}=$$

$$=\sum\frac{\left(\frac{1}{m_a+m_b-m_c}\right)^2}{a+b}\overset{BERGSTROM}{\ge}\frac{\left(\frac{1}{m_a+m_b-m_c}+\frac{1}{m_b+m_c-m_a}+\frac{1}{m_c+m_a-m_b}\right)^2}{2(a+b+c)}\ge$$

$$\overset{BERGSTROM}{\ge}\frac{81}{4s(m_a+m_b+m_c)^2}\overset{LEUENBERGER}{\ge}\frac{81}{4s(4R+r)^2}\ge$$

$$\overset{MITRINOVIC}{\ge}\frac{81}{\frac{4.3\sqrt{3}R}{2}.(4R+r)^2}\overset{EULER}{\ge}\frac{81}{\frac{4.3\sqrt{3}R}{2}.\frac{81R^2}{4}}=\frac{2\sqrt{3}}{9R^3}$$

Equality holds for $a=b=c$.