ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$a^a \cdot b^b \cdot c^c \cdot (m_a + m_b + m_c)^{2s} \geq (2s)^{2s} \cdot m_a^2 \cdot m_b^b \cdot m_c^c$$

Proposed by Daniel Sitaru – Romania

Solution by Tapas Das – India

$$a^a \cdot b^b \cdot c^c \cdot (m_a + m_b + m_c)^{2s} \geq (2s)^{2s} \cdot m_a^a \cdot m_b^b \cdot m_c^c$$

$$\left(\frac{a}{m_a}\right)^a \cdot \left(\frac{b}{m_b}\right)^b \cdot \left(\frac{c}{m_c}\right)^c \geq \left\{\frac{(a+b+c)}{m_a + m_b + m_c}\right\}^{2s}$$

$$\left(\frac{a}{m_a}\right)^a \cdot \left(\frac{b}{m_b}\right)^b \cdot \left(\frac{c}{m_c}\right)^c \geq \left(\frac{a+b+c}{m_a + m_b + m_c}\right)^{a+b+c}$$

GM $\geq$ HM

$$\left[\left(\frac{a}{m_a}\right)^a \cdot \left(\frac{b}{m_b}\right)^b \cdot \left(\frac{c}{m_c}\right)^c\right]^{\frac{1}{a+b+c}} \geq \frac{a+b+c}{\frac{a}{\left(\frac{a}{m_a}\right)} + \frac{b}{\left(\frac{b}{m_b}\right)} + \frac{c}{\left(\frac{c}{m_c}\right)}} = \frac{a+b+c}{m_a + m_b + m_c}$$

$$\therefore \left(\frac{a}{m_a}\right)^a \cdot \left(\frac{b}{m_b}\right)^b \cdot \left(\frac{c}{m_c}\right)^c \geq \left(\frac{a+b+c}{m_a + m_b + m_c}\right)^{a+b+c}$$

Equality holds for $a = b = c$.