ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$a\sqrt{4a^2+9b^2}+b\sqrt{4b^2+9c^2}+c\sqrt{4c^2+9a^2}\ge 10\sqrt{6}\cdot F$$

Proposed by Daniel Sitaru-Romania

Solution by Tapas Das-India

$$4a^2+9b^2\ge\frac{(2a+3b)^2}{2}$$

$$a\sqrt{4a^2+9b^2}+b\sqrt{4b^2+9c^2}+c\sqrt{4c^2+9a^2}\ge\sum\frac{a(2a+3b)}{\sqrt{2}}=$$

$$=\sqrt{2}\sum a^2+\frac{3}{\sqrt{2}}\sum ab\ge\sqrt{2}\sum ab+\frac{3}{\sqrt{2}}\sum ab\overset{Gordon}{\ge}$$

$$\ge\sqrt{2}\cdot 4\sqrt{3}\,F+\frac{3}{\sqrt{2}}\cdot 4\sqrt{3}\,F=4\sqrt{6}\,F+6\sqrt{6}\,F=10\sqrt{6}\,F$$

Equality holds for $a=b=c$.