ROMANIAN MATHEMATICAL MAGAZINE

In any Δ ABC, the following relationship holds :

$$\frac{m_a^2 m_b^2}{(a+b)a^2b^2}+\frac{m_b^2 m_c^2}{(b+c)b^2c^2}+\frac{m_c^2 m_a^2}{(c+a)c^2a^2}\ge\frac{9\sqrt{3}}{32R}$$

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Solution by Soumava Chakraborty-Kolkata-India

$$\sum_{cyc}\frac{m_a^2 m_b^2}{(a+b)a^2b^2}=\frac{81}{16}\sum_{cyc}\frac{\left(\frac{2}{3}m_a\right)^2\left(\frac{2}{3}m_b\right)^2}{(a+b)a^2b^2}=\frac{81}{16}\cdot\sum_{cyc}\frac{\frac{AG^2.BG^2}{a^2b^2}}{a+b}\overset{BERGSTROM}{\ge}$$

$$\ge\frac{81}{16}\cdot\frac{\left(\sum_{cyc}\frac{AG.BG}{ab}\right)^2}{a+b+b+c+c+a}\overset{HAYASHI}{\ge}\frac{81}{16}\cdot\frac{1}{4s}\overset{MITRINOVIC}{\ge}$$

$$\ge\frac{81}{16}\cdot\frac{1}{4R.\frac{3\sqrt{3}}{2}}=\frac{9\sqrt{3}}{32R}$$

$" = "$ iff Δ ABC is equilateral (QED)