ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$7+\cot^2 A+\cot^2 B+\cot^2 C\le 8\left(\frac{R}{2r}\right)^4$$

Proposed by George Apostolopoulos-Greece

Solution by Tapas Das-India

$$7+\cot^2 A+\cot^2 B+\cot^2 C=7+\sum\csc^2 A-3=4+\sum\frac{4R^2}{a^2}=$$

$$=4+4R^2\sum\frac{1}{a^2}\overset{Steining}{\le}4+\frac{4R^2}{4r^2}=(2)^2+\left(\frac{R}{r}\right)^2\overset{Euler}{\le}\left(\frac{R}{r}\right)^2+\left(\frac{R}{r}\right)^2=2\left(\frac{R}{r}\right)^2=$$

$$=2\cdot\frac{R^2R^2}{r^2R^2}\overset{Euler}{\le}\frac{2(R)^4}{r^2\cdot 4r^2}=\frac{8(R^4)}{16r^4}=8\left(\frac{R}{2r}\right)^4$$

$$Equality\ holds\ for\ A=B=C$$