ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$(\sin A+\sin B+\sin C)^2-\cos(A-B)-\cos(B-C)-\cos(C-A)\le\frac{15}{4}$$

Proposed by George Apostolopoulos-Greece

Solution by Tapas Das-India

$$\sum\sin A=\frac{s}{R}\ and\ \sum\cos(A-B)=\frac{s^2+r^2+2Rr}{2R^2}-1$$

$$(\sin A+\sin B+\sin C)^2-\cos(A-B)-\cos(B-C)-\cos(C-A)=$$

$$=\left(\frac{s}{R}\right)^2-\frac{s^2+r^2+2Rr}{2R^2}+1=$$

$$=\frac{s^2-r^2-2Rr}{2R^2}+1\overset{Gerrestn}{\le}\frac{4R^2+2Rr+2r^2}{2R^2}+1\overset{Euler}{\le}$$

$$\le\frac{4R^2+R^2+\frac{R^2}{2}}{2R^2}+1=\frac{11}{4}+1=\frac{15}{4}$$

$$Equality\ holds\ for\ A=B=C$$