ROMANIAN MATHEMATICAL MAGAZINE

If $x,y,z>0$ then in ΔABC the following relationship holds:

$$\frac{x}{y+z}\sin^2 A+\frac{y}{z+x}\sin^2 B+\frac{z}{x+y}\sin^2 C\geq\frac{9}{8}\left(\frac{R}{2r}\right)^{-2}$$

Proposed by George Apostolopoulos-Greece

Solution by Tapas Das-India

$$\frac{x}{y+z}\sin^2 A+\frac{y}{z+x}\sin^2 B+\frac{z}{x+y}\sin^2 C=$$

$$=\frac{1}{4R^2}\left(\frac{x}{y+z}a^2+\frac{y}{z+x}b^2+\frac{z}{x+y}c^2\right)\overset{Tsintsifas}{\geq}$$

$$\geq\frac{1}{4R^2}\,2\sqrt{3}\,F\overset{Mitrinovic}{\geq}\frac{1}{4R^2}\,2\sqrt{3}r.3\sqrt{3}r=\frac{9}{8}\left(\frac{2r}{R}\right)^2=\frac{9}{8}\left(\frac{R}{2r}\right)^{-2}$$

$$Equality\ for\ A=B=C\ and\ x=y=z.$$