ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{2}{R} \leq \frac{1}{s_a} + \frac{1}{s_b} + \frac{1}{s_c} \leq \frac{R}{2r^2}$$

Proposed by Marian Ursărescu-Romania

Solution by Tapas Das-India

$$s_a = \frac{2bcm_a}{b^2 + c^2}$$

$$m_a - s_a = m_a\left(1 - 2bc\frac{1}{b^2 + c^2}\right) = \frac{m_a}{b^2 + c^2}(b - c)^2 \geq 0 \ so\ \ m_a \geq \ s_a\ (1)$$

$$\frac{1}{s_a} + \frac{1}{s_b} + \frac{1}{s_c} \overset{Bergstrom}{\geq} \frac{(1 + 1 + 1)^2}{s_a + s_b + s_c} \overset{(1)}{\geq}. \frac{9}{m_a + m_b + m_c} \overset{Leuenberger}{\geq} \frac{9}{4R + r} \overset{Euler}{\geq} \frac{9}{\frac{9R}{2}} = \frac{2}{R}$$

$$\frac{1}{s_a} + \frac{1}{s_b} + \frac{1}{s_c} \leq \frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c} = \frac{1}{r} = \frac{r}{r^2} \overset{Euler}{\leq} \frac{\frac{R}{2}}{r^2} = \frac{R}{2r^2}$$

Equality for $a = b = c$.