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In ΔABC the following relationship holds:

$$\sum\left(\frac{a}{b}+\frac{b}{a}\right)\cos^2\frac{C}{2}\le\frac{3}{2}\left(1+\frac{R}{r}\right)$$

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Solution by Tapas Das-India

$$\sum\left(\frac{a}{b}+\frac{b}{a}\right)\cos^2\frac{C}{2}=\frac{1}{2}\sum\left(\frac{a}{b}+\frac{b}{a}\right)+\frac{1}{2}\sum\left(\frac{a}{b}+\frac{b}{a}\right)\cos C$$

$$\sum\left(\frac{a}{b}+\frac{b}{a}\right)\cos C=\sum\left(\frac{\sin A}{\sin B}+\frac{\sin B}{\sin A}\right)\cos C=\sum\left(\frac{\sin A\cos C}{\sin B}+\frac{\cos A\sin C}{\sin B}\right)=$$

$$=\sum\frac{\sin(A+C)}{\sin B}\overset{A+B+C=\pi}{=}\sum\frac{\sin B}{\sin B}=3$$

$$\sum\left(\frac{a}{b}+\frac{b}{a}\right)\cos^2\frac{C}{2}=\frac{1}{2}\sum\left(\frac{a}{b}+\frac{b}{a}\right)+\frac{1}{2}\sum\left(\frac{a}{b}+\frac{b}{a}\right)\cos C\overset{Bandila}{\le}\frac{1}{2}\cdot\frac{3R}{r}+\frac{3}{2}=\frac{3}{2}\left(1+\frac{R}{r}\right)$$

Equality holds for an equilateral triangle.