ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\cos\frac{A-B}{2}\cos\frac{B-C}{2}\cos\frac{C-A}{2}\geq 8\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

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Solution by Mirsadix Muefferov-Azerbaijan

$$\cos\frac{A-B}{2}\cos\frac{B-C}{2}\cos\frac{C-A}{2}\geq 8\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

$$\frac{\cos\frac{A-B}{2}}{\sin\frac{C}{2}}=\frac{2\cos\frac{A-B}{2}\cos\frac{C}{2}}{2\sin\frac{C}{2}\cos\frac{C}{2}}=\frac{2\cos\frac{A-B}{2}\sin\frac{A+B}{2}}{\sin C}=\frac{\sin A+\sin B}{\sin C}=$$

$$=\frac{2R\sin A+2R\sin B}{2R\sin C}=\frac{a+b}{c}\quad(1)$$

Analogously others:

$$\frac{\cos\frac{B-C}{2}}{\sin\frac{A}{2}}=\frac{b+c}{a}\quad(2)$$

$$\frac{\cos\frac{C-A}{2}}{\sin\frac{B}{2}}=\frac{a+c}{b}\quad(3)$$

If we multipy $(1),(2)$ *and* (3) *side by side we have* ,

$$\frac{\cos\frac{A-B}{2}}{\sin\frac{C}{2}}\cdot\frac{\cos\frac{B-C}{2}}{\sin\frac{A}{2}}\cdot\frac{\cos\frac{C-A}{2}}{\sin\frac{B}{2}}=\frac{a+b}{c}\cdot\frac{b+c}{a}\cdot\frac{a+c}{b}\quad(4)$$

(4) *from here* :

$$\frac{a+b}{c}\cdot\frac{b+c}{a}\cdot\frac{a+c}{b}\overset{A-G}{\underbrace{\geq}}\frac{2\sqrt{ab}}{c}\cdot\frac{2\sqrt{bc}}{a}\cdot\frac{2\sqrt{ac}}{b}=8\ (QED)$$

Equality holds : $a=b=c$