

ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{\sin A}{2 + \cos A} + \frac{\sin B}{2 + \cos B} + \frac{\sin C}{2 + \cos C} \leq \frac{3\sqrt{3}}{5}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$\text{Let } f(x) = \frac{\sin x}{2 + \cos x}, x \in (0, \pi). f'(x) = \frac{2\cos x}{(2 + \cos x)^2},$$
$$f''(x) = \frac{2\sin x(\cos x - 2)}{(2 + \cos x)^3} < 0 \text{ so } f \text{ is concave in } (0, \pi)$$

Now using this result

$$\frac{\sin A}{2 + \cos A} + \frac{\sin B}{2 + \cos B} + \frac{\sin C}{2 + \cos C} \stackrel{\text{Jensen}}{\leq} 3 \cdot \frac{\sin\left(\frac{\pi}{3}\right)}{2 + \cos\left(\frac{\pi}{3}\right)} = \frac{3\sqrt{3}}{5}.$$

$$\text{Equality for } A = B = C = \frac{\pi}{3}$$