ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$a\tan\frac{A}{2}+b\tan\frac{B}{2}+c\tan\frac{C}{2}\geq 6r$$

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WLOG: $a\leq b\leq c\Rightarrow A\leq B\leq C\Rightarrow \tan\frac{A}{2}\leq \tan\frac{B}{2}\leq \tan\frac{C}{2}$

$$a\tan\frac{A}{2}+b\tan\frac{B}{2}+c\tan\frac{C}{2}\overset{CEBYSHEV}{\geq}\frac{1}{3}(a+b+c)\sum_{cyc}\tan\frac{A}{2}=$$

$$=\frac{2s}{3}\sum_{cyc}\tan\frac{A}{2}\overset{MITRINOVIC}{\geq}\frac{2\cdot 3\sqrt{3}r}{3}\sum_{cyc}\tan\frac{A}{2}\overset{JENSEN}{\geq}$$

$$\geq 2\sqrt{3}r\cdot 3\tan\left(\frac{\frac{A}{2}+\frac{B}{2}+\frac{C}{2}}{3}\right)=6\sqrt{3}r\cdot\tan\frac{A+B+C}{6}=$$

$$=6\sqrt{3}r\cdot\tan\frac{\pi}{6}=6\sqrt{3}r\cdot\frac{\sqrt{3}}{3}=6r$$

Equality holds for $a=b=c$.