ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$F \leq \frac{1}{\sqrt{3}} \sqrt[3]{(m_a^2 m_b^2 m_c^2)}$$

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Lemma: In ΔABC the following relationship holds:

$$m_a \geq \sqrt{s(s-a)}$$

Proof (by editor):

$$m_a \geq \sqrt{s(s-a)} \Leftrightarrow m_a^2 \geq \frac{a+b+c}{2} \cdot \frac{b+c-a}{2} \Leftrightarrow$$

$$\frac{b^2+c^2}{2} - \frac{a^2}{4} \geq \frac{b^2+c^2+2bc-a^2}{4} \Leftrightarrow 2b^2+2c^2 \geq b^2+c^2+2bc$$

$$b^2+c^2-2bc \geq 0 \Leftrightarrow (b-c)^2 \geq 0$$

Back to the problem:

By lemma:

$$\prod m_a^2 \geq s^3(s-a)(s-b)(s-c) = s^4 r^2 \overset{Mitrinovic}{\geq} s^3 . 3\sqrt{3}\, r^3$$

$$\frac{1}{\sqrt{3}} \sqrt[3]{(m_a^2 m_b^2 m_c^2)} \geq \frac{1}{\sqrt{3}} \sqrt[3]{(s^3 . 3\sqrt{3}\, r^3)} = r.s = F$$

Equality holds for $a = b = c$.