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In $\triangle ABC$ the following relationship holds:

$$a^2 \cos \frac{A}{2} + b^2 \cos \frac{B}{2} + c^2 \cos \frac{C}{2} \leq \frac{9\sqrt{3}}{2} R^2$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$\begin{aligned} \sum \cos \frac{A}{2} &\stackrel{JENSEN}{\leq} 3 \cos \frac{\pi}{6} = \frac{3\sqrt{3}}{2} \\ a^2 \cos \frac{A}{2} + b^2 \cos \frac{B}{2} + c^2 \cos \frac{C}{2} &\stackrel{CEBYSHEV}{\leq} \frac{1}{3} \left(\sum a^2 \right) \left(\sum \cos \frac{A}{2} \right) \leq \\ &\stackrel{LEIBNIZ}{\leq} \frac{1}{3} 9R^2 \frac{3\sqrt{3}}{2} = \frac{9\sqrt{3}}{2} R^2 \end{aligned}$$

Equality holds for $a = b = c$.