ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\cos A+\cos B+\cos C+\frac{4}{\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}\geq\frac{67}{2}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$\cos A+\cos B+\cos C+\frac{4}{\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}=$$

$$=1+\frac{r}{R}+\frac{4}{\frac{r}{4R}}=1+\frac{1}{x}+16x\left(where\,\frac{R}{r}=x\geq 2\ Euler\right)$$

$we\ need\ to\ show:$

$$1+\frac{1}{x}+16x\geq\frac{67}{2}\ or$$

$$32x^2-65x+2\geq 0\ or$$

$$(32x-1)(x-2)\geq 0\ true\ as\ x\geq 2$$

Equality holds for $a=b=c$.